

**Exercise 38**

Let  $f(x) = \log_b(3x^2 - 2)$ . For what value of  $b$  is  $f'(1) = 3$ ?

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**Solution**

Start by taking the derivative of the function with respect to  $x$ .

$$\begin{aligned} f'(x) &= \frac{d}{dx} [\log_b(3x^2 - 2)] \\ &= \frac{1}{(3x^2 - 2) \ln b} \cdot \frac{d}{dx} (3x^2 - 2) \\ &= \frac{1}{(3x^2 - 2) \ln b} \cdot (6x) \\ &= \frac{6x}{(3x^2 - 2) \ln b} \end{aligned}$$

Set  $x = 1$  to get  $f'(1)$ .

$$f'(1) = \frac{6(1)}{[3(1)^2 - 2] \ln b} = \frac{6}{\ln b}$$

In order for  $f'(1) = 3$ ,

$$\frac{6}{\ln b} = 3$$

$$\frac{\ln b}{6} = \frac{1}{3}$$

$$\ln b = 2$$

$$b = e^2.$$